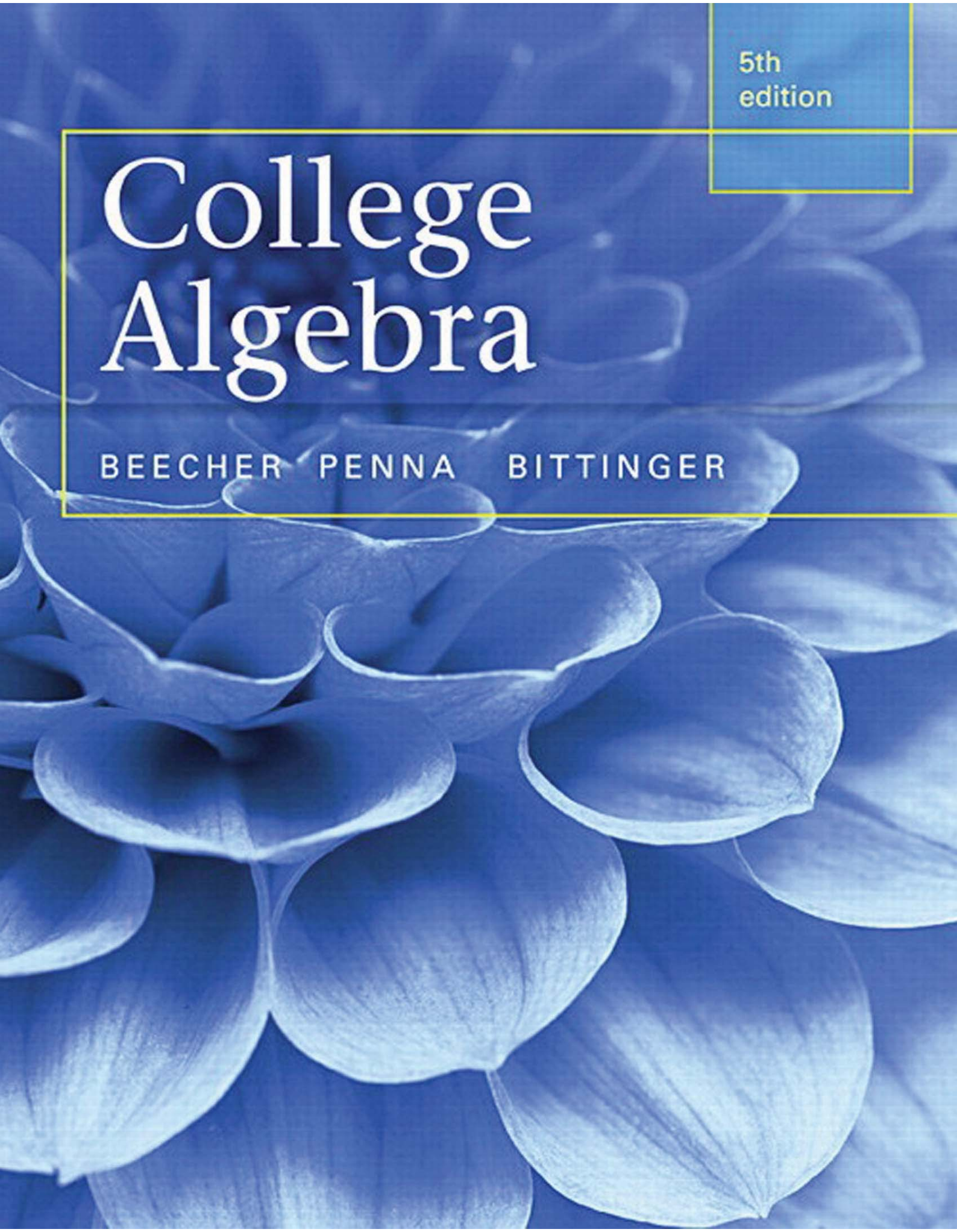

5th
edition
College
Algebra
BEECHER PENNA BITTINGER

College Algebra

This page intentionally left blank

EDITION

5

College Algebra

Judith A. Beecher

Judith A. Penna

Marvin L. Bittinger

Indiana University Purdue University Indianapolis

PEARSON

Boston Columbus Hoboken Indianapolis New York San Francisco
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montréal Toronto
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Editorial Director	Chris Hoag
Editor in Chief	Anne Kelly
Sponsoring Editor	Kathryn O'Connor
Editorial Assistant	Judith Garber
Program Manager	Tatiana Anacki
Project Manager	Kathleen A. Manley
Program Management Team Lead	Marianne Stepanian
Project Management Team Lead	Christina Lepre
Media Producer	Erica Lange
TestGen Content Manager	Marty Wright
MathXL Content Manager	Kristina Evans
Marketing Manager	Peggy Lucas
Marketing Assistant	Justine Goulart
Senior Author Support/ Technology Specialist	Joe Vetere
Rights and Permissions Project Manager	Diahanne Lucas Dowridge
Procurement Specialist	Carol Melville
Associate Director of Design	Andrea Nix
Program Design Lead and Cover Design	Barbara T. Atkinson
Text Design, Art Editing, and Photo Research	The Davis Group, Inc.
Editorial and Production Coordination	Martha Morong/Quadrata, Inc.
Composition	Lumina Datamatics Ltd.
Illustrations	Network Graphics, William Melvin, and Lumina Datamatics Ltd.
Cover Image	Petals of a flower, © irin-k/ Shutterstock

Library of Congress Cataloging-in-Publication Data

Beecher, Judith A.
College algebra / Judith A. Beecher, Judith A. Penna, Marvin L. Bittinger, Indiana University Purdue University Indianapolis, 5th edition.
pages cm
Includes indexes.
ISBN 0-321-96957-X (student edition)
1. Algebra—Textbooks. I. Penna, Judith A. II. Bittinger, Marvin L. III. Title.
QA152.3.B44 2016
512.9dc23

2014022118

1 2 3 4 5 6 7 8 9 10—DOW—18 17 16 15 14

PEARSON

www.pearsonhighered.com

SE ISBN-13: 978-0-321-96957-6
ISBN-10: 0-321-96957-X

Contents

Preface

This *College Algebra* textbook is known for enabling students to "see the math" through its

- focus on visualization,
- early introduction of functions,
- complete, optional technology coverage, and
- connections between math concepts and the real world.

New!

With the new edition, we continue to innovate by positioning the review material as a more effective tool for teachers and students. Chapter R from the previous edition has been condensed into 25 Just-In-Time review topics that are placed at the back of the book. This new review feature is designed to give each student the opportunity to be successful in this course by providing a quick review of topics from intermediate algebra that will be built upon in new college algebra topics. The review can be used in an individualized instruction format since some students will require more review than others. Treating the review in this manner will allow more time to cover the college algebra topics in the syllabus.

On the other hand, some instructors might choose to review some or all of the topics with the entire class at the beginning of the course or in a just-in-time format as each is needed. We think instructors will appreciate the flexibility that the Just-In Time feature offers them.

Additional resources in the MyMathLab courses reflect the themes of just-in-time review and concept retention. For example, new Cumulative Review assignments allow students to synthesize and retain concepts learned throughout the course.

Our overarching goal is to provide students with a learning experience that will not only lead to success in this course, but also prepare them to be successful in the mathematics courses they take in the future.

▶ Content Changes to the Fifth Edition

JUST IN TIME 10

- **Just-In-Time Review** Review of prerequisite algebra topics is now presented when students need it most.
  - A set of 25 numbered short review topics creates an efficient review of intermediate algebra topics.
  - This feature is placed at the back of the text. Just-In-Time icons are positioned throughout the text next to the example where review of an intermediate algebra topic would be helpful.
  - Even more just-in-time review resources are available in the MyMathLab course for *College Algebra with Integrated Review* and in the Getting Ready MyMathLab exercises.
- **Informed Exercises** We have analyzed the MyMathLab usage data, which has helped us revise our exercises for this new edition. The goal is to ultimately improve the quality and quantity of exercises that matter the most to instructors and students.
- **Symmetry and Transformations** These topics are now presented in two sections rather than one.

▶ Emphasis on Functions

Functions are the core of this course and are presented as a thread that runs throughout the course rather than as an isolated topic. We introduce functions in Chapter 1, whereas many traditional college algebra textbooks cover equation-solving in Chapter 1. Our approach of introducing students to a relatively new concept at the beginning of the course, rather than requiring them to begin with a review of material that was previously covered in intermediate algebra, immediately engages them and serves to help them avoid the temptation to not study early in the course because "I already know this."

The concept of a function can be challenging for students. By repeatedly exposing them to the language, notation, and use of functions, demonstrating visually how functions relate to equations and graphs, and also showing how functions can be used to model real data, we hope to ensure that students not only become comfortable with functions but also come to understand and appreciate them. You will see this emphasis on functions woven throughout the other themes that follow.

Classify the Function Exercises With a focus on conceptual understanding, students are asked periodically to identify a number of functions by their type (linear, quadratic, rational, and so on). As students progress through the text, the variety of functions with which they are familiar increases and these exercises become more challenging. The "classifying the function" exercises appear with the review exercises in the Skill Maintenance portion of an exercise set. (See pp. 266 and 356.)

▶ Visual Emphasis

Our early introduction of functions allows graphs to be used to provide a visual aspect to solving equations and inequalities. For example, we are able to show students both algebraically and visually that the solutions of a quadratic equation $ax^2 + bx + c = 0$ are the zeros of the quadratic function $f(x) = ax^2 + bx + c$, as well as the first coordinates of the x-intercepts of the graph of that function. This makes it possible for students, particularly visual learners, to gain a quick understanding of these concepts. (See pp. 182, 185, 227, 285, and 344.)

Visualizing the Graph Appearing at least once in every chapter, this feature provides students with an opportunity to match an equation with its graph by focusing on the characteristics of the equation and the corresponding attributes of the graph. (See pp. 143, 198, and 280.) In addition to this full-page feature, many of the exercise sets include exercises in which the student is asked to match an equation with its graph or to find an equation of a function from its graph. (See pp. 145, 146, 236, and 330.) In MyMathLab, animated Visualizing the Graph features for each chapter allow students to interact with graphs on an entirely new level.

Side-by-Side Examples Many examples are presented in a side-by-side, two-column format in which the algebraic solution of an equation appears in the left column and a graphical solution appears in the right column. (See pp. 176, 290–291, 360, and 361.) This enables students to visualize and comprehend the connections among the solutions of an equation, the zeros of a function, and the x-intercepts of the graph of a function.

Technology Connections This feature appears throughout the text to demonstrate how a graphing calculator can be used to solve problems. The technology is set apart from the traditional exposition so that it does not intrude if no technology is desired. Although students might not be using graphing calculators, the graphing calculator windows that appear in the Technology Connection features enhance the visual element of the text, providing graphical interpretations of solutions of equations, zeros of functions, and x-intercepts of graphs of functions. (See pp. 21, 181, and 360.) A graphing calculator manual providing keystroke-level instruction, written by author Judy Penna, is available online.

Making Connections

Zeros, Solutions, and *x*-Intercepts We find that when students understand the connections among the real zeros of a function, the solutions of its associated equation, and the first coordinates of the *x*-intercepts of its graph, a door opens to a new level of mathematical comprehension that increases the probability of success in this course. We emphasize zeros, solutions, and *x*-intercepts throughout the text by using consistent, precise terminology and including exceptional graphics. Seeing this theme repeated in different contexts leads to a better understanding and retention of these concepts. (See pp. 176 and 185.)

Connecting the Concepts This feature highlights the importance of connecting concepts. When students are presented with concepts in visual form—using graphs, an outline, or a chart—rather than merely in paragraphs of text, comprehension is streamlined and retention is enhanced. The visual aspect of this feature invites students to stop and check their understanding of how concepts work together in one section or in several sections. This check in turn enhances student performance on homework assignments and exams. (See pp. 73, 185, and 253.)

Annotated Examples We have included over 730 annotated examples designed to fully prepare the student to work the exercises. Learning is carefully guided with the use of numerous color-coded art pieces and step-by-step annotations. Substitutions and annotations are highlighted in red for emphasis. (See pp. 179 and 352.)

Now Try Exercises Now Try Exercises are found after nearly every example. This feature encourages active learning by asking students to do an exercise in the exercise set that is similar to the example the student has just read. (See pp. 182, 272, and 328.)

Synthesis Exercises These exercises appear at the end of each exercise set and encourage critical thinking by requiring students to synthesize concepts from several sections or to take a concept a step further than in the general exercises. For the Fifth Edition, these exercises are assignable in MyMathLab. (See pp. 32, 255, 333, and 380.)

Real-Data Applications We encourage students to see and interpret the mathematics that appears every day in the world around them. Throughout the writing process, we conducted an energetic search for real-data applications, and the result is a variety of examples and exercises that connect the mathematical content with everyday life. Most of these applications feature source lines and many include charts and graphs. Many are drawn from the fields of health, business and economics, life and physical sciences, social science, and areas of general interest such as sports and travel. (See pp. 39 ("Food Stamp Program"), 66 ("Words in Languages"), 133 ("Peace Corps Volunteers"), 187 ("Funding for Afghan Security"), 236 ("Vinyl Album Sales"), 331 ("Alternative-Fuel Vehicles"), 406 ("Cosmetic Surgery"), 415 ("Top Auction Art Sales"), 494 ("The Ellipse at the White House"), and 546 ("The Economic Multiplier; Super Bowl XLVII").)

Ongoing Review

The most significant change to the Fifth Edition is the new Just-in-Time Review feature, designed to provide students with efficient and effective review of basic algebra skills.

JUST IN TIME 10

New! Just-in-Time Review Chapter R has been condensed into 25 numbered short review topics to create an efficient review of intermediate algebra topics. This feature is placed at the back of the book.

- Just-In-Time icons are placed throughout the text next to the example where review of an intermediate algebra topic would be helpful. (See pp. 35, 99, 115, 171, 232, and 319.)
- The coverage of each topic contains worked-out examples and a short exercise set. Answers to all exercises appear at the back of the book.

- Worked-out solutions to all exercises are included in the *Student Solutions Manual*.
- Students can find additional review support in the MyMathLab course for College Algebra with Integrated Review and in the Getting Ready MyMathLab exercises.

Mid-Chapter Mixed Review This review reinforces understanding of the mathematical concepts and skills covered in the first half of the chapter before students move on to new material in the second half of the chapter. Each review begins with at least three true/false exercises that require students to consider the concepts they have studied and also contains exercises that drill the skills from all prior sections of the chapter. These exercises are assignable in MyMathLab. (See pp. 125–126 and 256–257.)

Collaborative Discussion and Writing Exercises appear in the Mid-Chapter Mixed Review as well. These exercises can be discussed in small groups or by the class as a whole to encourage students to talk about the key mathematical concepts in the chapter. They can also be assigned to individual students to give them an opportunity to write about mathematics. (See pp. 202 and 257.)

A section reference is provided for each exercise in the Mid-Chapter Mixed Review. This tells the student which section to refer to if help is needed to work the exercise. Answers to all exercises in the Mid-Chapter Mixed Review are given at the back of the book.

Study Guide This feature is found at the beginning of the **Summary and Review** near the end of each chapter. Presented in a two-column format and organized by section, this feature gives key concepts and terms in the left column and a worked-out example in the right column. It provides students with a concise and effective review of the chapter that is a solid basis for studying for a test. In MyMathLab, these Study Guides are accompanied by narrated examples to reinforce the key concepts and ideas. (See pp. 214–220 and 381–387.)

Exercise Sets There are over 5040 exercises in this text. The exercise sets are enhanced with real-data applications and source lines, detailed art pieces, tables, graphs, and photographs. In addition to the exercises that provide students with concepts presented in the section, the exercise sets feature the following elements to provide ongoing review of topics presented earlier:

- **Skill Maintenance Exercises.** These exercises provide an ongoing review of concepts previously presented in the course, enhancing students' retention of these concepts. These exercises include **Vocabulary Reinforcement**, described below, and **Classifying the Function** exercises, described earlier in the section "Emphasis on Functions." A section reference is provided for each exercise. This tells the student which section to refer to if help is needed to work the exercise. Answers to all Skill Maintenance exercises appear in the answer section at the back of the book. (See pp. 133, 210, 283, and 347.)
- **Enhanced Vocabulary Reinforcement Exercises.** This feature checks and reviews students' understanding of the vocabulary introduced throughout the text. It appears once in every chapter, in the Skill Maintenance portion of an exercise set, and is intended to provide a continuing review of the terms that students must know in order to be able to communicate effectively in the language of mathematics. (See pp. 84, 154, 214, and 283.) These are now assignable in MyMathLab and can serve as reading quizzes.
- **Enhanced Synthesis Exercises.** These exercises are described under the Making Connections heading and are also assignable in MyMathLab.

Review Exercises These exercises in the **Summary and Review** supplement the Study Guide by providing a thorough and comprehensive review of the skills taught in the chapter. A group of true/false exercises appears first, followed by a large number of exercises that drill the skills and concepts taught in the chapter. In addition, three

multiple-choice exercises, one of which involves identifying the graph of a function, are included in the Review Exercises for every chapter. Each review exercise is accompanied by a section reference that, as in the Mid-Chapter Mixed Review, directs students to the section in which the material being reviewed can be found. Collaborative Discussion and Writing exercises are also included. These exercises are described under the Mid-Chapter Mixed Review heading on p. xiv. (See pp. 220–223 and 388–390.)

Chapter Test The test at the end of each chapter allows students to test themselves and target areas that need further study before taking the in-class test. Each Chapter Test includes a multiple-choice exercise involving identifying the graph of a function. Answers to all questions in the Chapter Tests appear in the answer section at the back of the book, along with corresponding section references. (See pp. 223–224 and 391–392.)

DOMAIN

REVIEW SECTION 1.2

Review Icons Placed next to the concept that a student is currently studying, a review icon references a section of the text in which the student can find and review topics on which the current concept is built. (See pp. 267 and 324.)

Acknowledgments

We wish to express our heartfelt thanks to a number of people who have contributed in special ways to the development of this textbook. Our editor, Kathryn O'Connor, encouraged and supported our vision. We are very appreciative of the marketing insight provided by Peggy Lucas, our marketing manager, and of the support that we received from the entire Pearson team, including Kathy Manley, project manager, Barbara Atkinson, cover designer, Judith Garber, editorial assistant, and Justine Goulart, marketing assistant. We also thank Erica Lange, media producer, for her creative work on the media products that accompany this text. And we are immensely grateful to Martha Morong for her editorial and production services, and to Geri Davis for her text design and art editing, and for the endless hours of hard work they have done to make this a book of which we are proud. We also thank Mike Rosenborg for his meticulous accuracy checking and proofreading of the text.

The following reviewers made invaluable contributions to the development of the Fifth Edition and we thank them for that:

Holly Ashton, *Pikes Peak Community College*
Stacie Bardran, *Embry-Riddle Aeronautical University*
Kim Berges, *Morrisville State College*
Sherry S. Biggers, *Clemson University Department of Mathematical Sciences*
Nadine Bluett, *Front Range Community College*
Gary Brice, *Lamar University*
Christine Bush, *Palm Beach Community College, Lake Worth*
Shawn Clift, Ph.D., *Eastern Kentucky University*
Walter Czarnec, *Framingham State College*
Joseph De Guzman, *Riverside College, Norco Campus*
Douglas Dunbar, *Okaloosa-Walton Community College*
Wayne Ferguson, *Northwest Mississippi Community College*
Joseph Gaskin, *State University of New York, Oswego*
Sunshine Gibbons, *Southeast Missouri State University*
Dauhrice K. Gibson, (retired), *Gulf Coast Community College*
Jim Graziose, *Palm Beach Community College*
Joseph Lloyd Harris, *Gulf Coast Community College*
Dr. Mako E. Haruta, *University of Hartford*
Susan K. Hitchcock, *Palm Beach Community College*
Sharon S. Hudson, *Gulf Coast Community College*
Patricia Ann Hussey, *Triton College*
Jennifer Jameson, *Coconino Community College, Flagstaff*

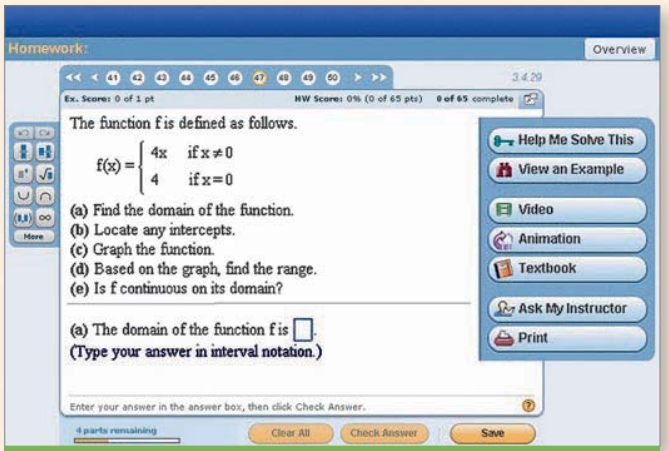
Cheryl Kane, *University of Nebraska*
Marjorie S. LaSalle, *DeKalb College*
Valerie LaVoice, *Community College System of New Hampshire*
Susan Leland, *Montana Tech of the University of Montana*
Jeremy Lyle, *The University of Southern Mississippi*
Bernard F. Mathon, *Miami-Dade College*
Debra McCandrew, *Florence-Darlington Technical College*
Barry J. Monk, *Macon State College*
Claude Moore, *Cape Fear Community College*
Darla Ottman, *Elizabethtown Community College*
Vicki Partin, *Lexington Community College*
Martha Pate, *Lamar State College—Port Arthur*
Leslie Richardson, *College of the Mainland*
Kathy V. Rodgers, *University of Southern Indiana*
Lucille Roth, *Tech of the Low Country*
Abdelrida Saleh, *Miami-Dade College*
Nicholas Sedlock, *Framingham State University*
Pavel Sikorskii, *Michigan State University*
Russell Simmons, *Brookhaven College*
Rajalakshmi Sriram, *Okaloosa-Walton Community College*
Corwin Stanford, *The University of Southern Mississippi*

J.A.B.
J.A.P.
M.L.B.

BREAKTHROUGH
To improving results

Get the most out of MyMathLab®

MyMathLab creates personalized experiences to help each student achieve success and provides powerful tools so instructors can create the perfect learning experiences for their courses.



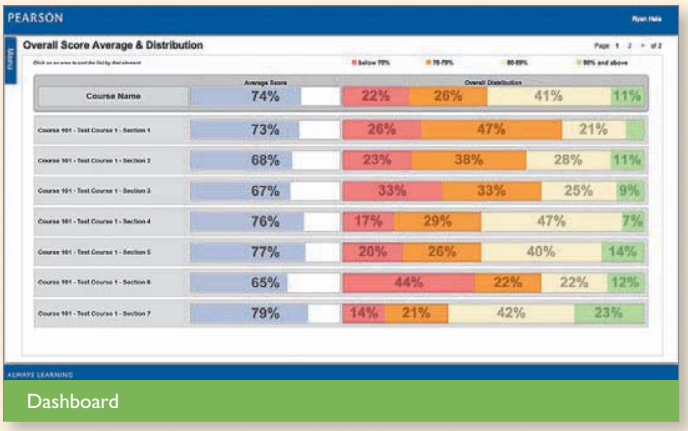
Exercise with feedback and learning aids

Personalized Support *for* Students

- MyMathLab comes with many learning resources—eText, animations, videos, and more—all designed to support you as you complete your assignments.
- Whether you're doing homework or working from the adaptive study plan, you'll receive immediate feedback, so you'll know exactly where you need help.

Data-Driven Reporting *for* Instructors

- MyMathLab's comprehensive online gradebook automatically tracks students' results on tests, quizzes, homework, and in the study plan.
- The Reporting Dashboard makes it easier than ever to identify topics where students are struggling or specific students who may need extra help.



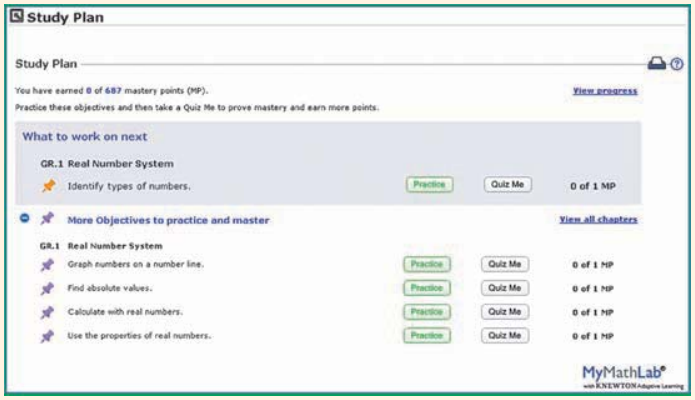
Dashboard

www.mymathlab.com

Resources for Success

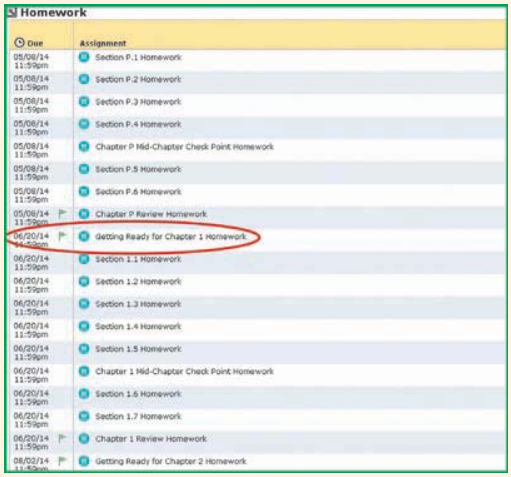
MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from an **experienced partner** with educational expertise and an eye on the future. **MyMathLab** helps prepare students and gets them thinking more conceptually and visually through the following features:



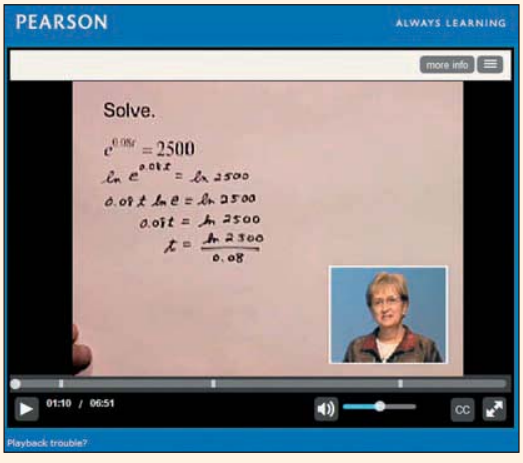
◄ Adaptive Study Plan

The Study Plan makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized content–reinforcing concepts that target each student's strengths and weaknesses.



Getting Ready ►

Students refresh prerequisite topics through assignable skill review quizzes and personalized homework integrated within **MyMathLab**.

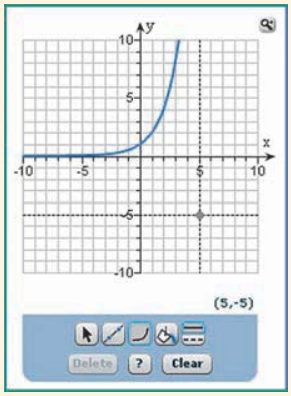


◄ Video Assessment

Video assessment is tied to key author example videos to check students' conceptual understanding of important math concepts.

Enhanced Graphing Functionality ►

New functionality within the graphing utility allows graphing of 3-point quadratic functions, 4-point cubic graphs, and transformations in exercises.



Skills for Success Modules are integrated within the **MyMathLab** course to help students succeed in collegiate courses and prepare for future professions.

Skill Maintenance These exercises support ongoing review at the course level and help students maintain essential skills.

Instructor Resources

Additional resources can be downloaded from **www.pearsonhighered.com** or hardcopy resources can be ordered from your sales representative.

Ready to Go MyMathLab® Course

Now it is even easier to get started with **MyMathLab**. The Ready to Go **MyMathLab** course option includes author-chosen preassigned homework, integrated review, and more.

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

Feature presentations written and designed specifically for this text. These lecture slides provide an outline for presenting definitions, figures, and key examples from the text.

Annotated Instructor's Edition

Includes all answers to the exercise sets, usually on the page on which the exercises appear. Sample homework assignments are indicated by a blue underline within each end-of-section exercise set and may be assigned in MyMathLab.

Instructor's Solutions Manual (Download Only)

Written by Judy Penna, this resource contains worked-out solutions to all exercises in the exercise sets, Mid-Chapter Mixed Reviews, Chapter Reviews, and Chapter Tests, as well as solutions for all the Just-In-Time exercises.

Online Test Bank (Download Only)

Contains four free-response text forms for each chapter following the same format and having the same level of difficulty as the test in the main text and two multiple-choice test forms for each chapter. It also provides six forms of the final examination, four with free-response questions and two with multiple-choice questions.

Student Resources

Additional resources to help student success.

Author Example Videos

Ideal for distance learning or supplemental instruction, these videos feature authors Judy Beecher and Judy Penna working through and explaining examples in the text. Assignable in MyMathLab with new Video Assessment questions.

New! Video Notebook

The new Video Notebook contains fill-in-the-blank worksheets to accompany the video examples presented by the authors. Key definitions, theorems, and procedures are also included. After filling in the worksheet while watching the video, the student has an excellent study guide for review and test preparation. This is available in print or as a PDF or Word document in MyMathLab.

Student's Solutions Manual

Written by author Judy Penna, this resource contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the exercise sets, Mid-Chapter Mixed Reviews, and Chapter Reviews, as well as solutions for all the Chapter Test exercises and the Just-In-TIme exercises.

Graphing Calculator Manual

Contains keystroke level instruction for the Texas Instruments TI-84 Plus using MathPrint OS. Mirrors the topic order in the main text to provide a just-in-time mode of instruction.

To the Student

GUIDE TO SUCCESS

Success can be planned. Combine goals and good study habits to create a plan for success that works for you. The following list contains study tips that your authors consider most helpful.

Skills for Success

- **Set goals and expect success.** Approach this class experience with a positive attitude.
- **Communicate with your instructor** when you need extra help.
- **Take your text with you to class and lab.** Each section in the text is designed with headings and boxed information that provide an outline for easy reference.
- **Ask questions in class, lab, and tutoring sessions.** Instructors encourage them, and other students probably have the same questions.
- **Begin each homework assignment as soon as possible.** If you have difficulty, you will then have the time to access supplementary resources.
- **Carefully read the instructions** before working homework exercises **and include all steps.**
- **Form a study group** with fellow students. Verbalizing questions about topics that you do not understand can clarify the material for you.
- After each quiz or test, **write out corrected step-by step solutions** to all missed questions. They will provide a valuable study guide for the midterm exam and the final exam.
- **MyMathLab has numerous tools to help you succeed.** Use MyMathLab to create a personalized study plan and practice skills with sample quizzes and tests.
- **Knowing math vocabulary is an important step toward success.** Review vocabulary with Vocabulary Reinforcement exercises in the text and in MyMathLab.
- If you miss a lecture, **watch the video in the Multimedia Library** of MyMathLab that explains the concepts you missed.

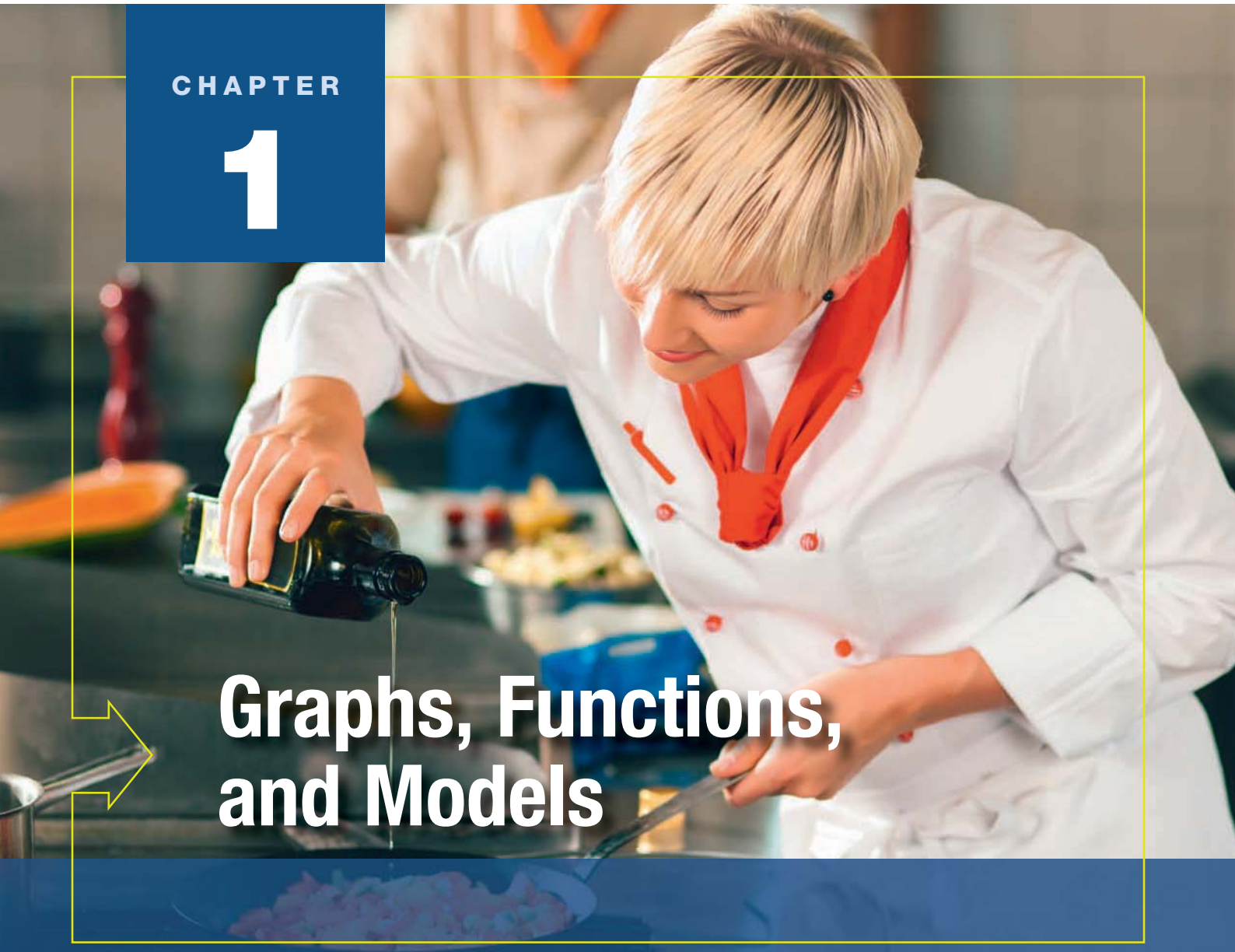
In writing this textbook, we challenged ourselves to do everything possible to help you learn the concepts and skills contained between its covers so that you will be successful in this course and in the mathematics courses you take in the future. We realize that your time is both valuable and limited, so we communicate in a highly visual way that allows you to learn quickly and efficiently. We are confident that, if you invest an adequate amount of time in the learning process, this text will be of great value to you. We wish you a positive learning experience.

Judy Beecher
Judy Penna
Marv Bittinger

CHAPTER

1

Graphs, Functions, and Models



APPLICATION

This problem appears as Exercise 67 in Exercise Set 1.5.

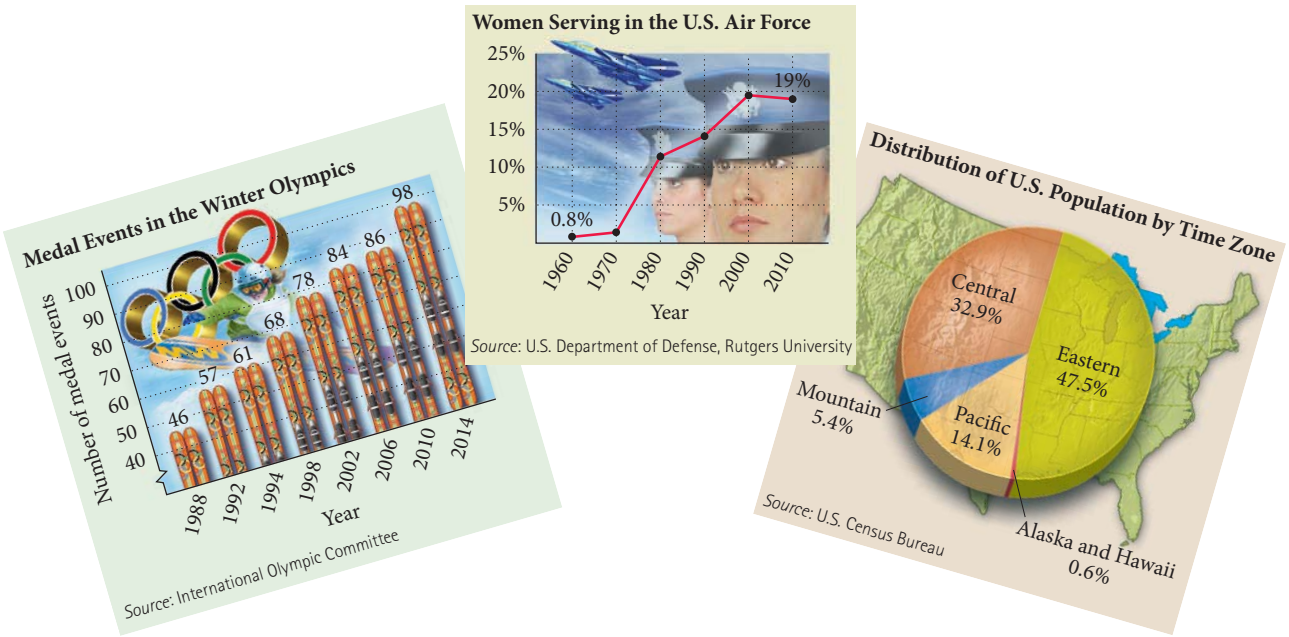
Together, Italy, Spain, and the United States consume 58% of the world's olive oil. The percentage consumed in Italy is $3\frac{3}{4}$ times the percentage consumed in the United States. The percentage consumed in Spain is $\frac{2}{3}$ of the percentage consumed in Italy. (*Source*: www.OliveOilEmporium.com) Find the percent of the world's olive oil consumed in each country.

1.1 Introduction to Graphing

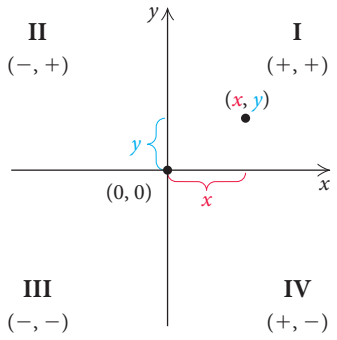
- Plot points.
- Determine whether an ordered pair is a solution of an equation.
- Find the x- and y-intercepts of an equation of the form $Ax + By = C$.
- Graph equations.
- Find the distance between two points in the plane and find the midpoint of a segment.
- Find an equation of a circle with a given center and radius, and given an equation of a circle in standard form, find the center and the radius.
- Graph equations of circles.

Graphs

Graphs provide a means of displaying, interpreting, and analyzing data in a visual format. It is not uncommon to open a newspaper or a magazine and encounter graphs. Examples of bar, line, and circle graphs are shown below.



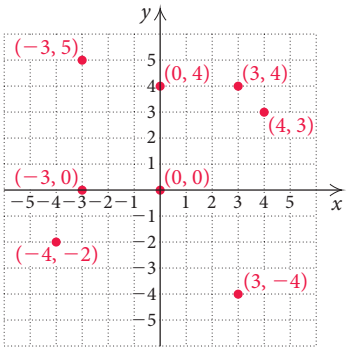
Many real-world situations can be modeled, or described mathematically, using equations in which two variables appear. We use a plane to graph a pair of numbers. To locate points on a plane, we use two perpendicular number lines, called **axes**, that intersect at (0, 0). We call this point the **origin**. The horizontal axis is called the ***x*-axis**, and the vertical axis is called the ***y*-axis**. (Other variables, such as a and b, can also be used.) The axes divide the plane into four regions,



called **quadrants**, denoted by Roman numerals and numbered counterclockwise from the upper right. Arrows show the positive direction of each axis.

Each point (x, y) in the plane is described by an **ordered pair**. The first number, x, indicates the point's horizontal location with respect to the y-axis, and the second number, y, indicates the point's vertical location with respect to the x-axis. We call x the **first coordinate,** the ***x*-coordinate**, or the **abscissa**. We call y the **second coordinate,** the ***y*-coordinate**, or the **ordinate**. Such a representation is called the **Cartesian coordinate system** in honor of the French mathematician and philosopher René Descartes (1596–1650).

In the first quadrant, both coordinates of a point are positive. In the second quadrant, the first coordinate is negative and the second is positive. In the third quadrant, both coordinates are negative, and in the fourth quadrant, the first coordinate is positive and the second is negative.



EXAMPLE 1 Graph and label the points $(-3, 5)$, $(4, 3)$, $(3, 4)$, $(-4, -2)$, $(3, -4)$, $(0, 4)$, $(-3, 0)$, and $(0, 0)$.

Solution To graph or **plot** $(-3, 5)$, we note that the x-coordinate, -3, tells us to move from the origin 3 units horizontally in the negative direction, or 3 units to the left of the y-axis. Then we move 5 units up from the x-axis.* To graph the other points, we proceed in a similar manner. (See the graph at left.) Note that the point $(4, 3)$ is different from the point $(3, 4)$.

Now Try Exercise 3.

Solutions of Equations

Equations in two variables, like $2x + 3y = 18$, have solutions (x, y) that are ordered pairs such that when the first coordinate is substituted for x and the second coordinate is substituted for y, the result is a true equation. The first coordinate in an ordered pair generally represents the variable that occurs first alphabetically.

EXAMPLE 2 Determine whether each ordered pair is a solution of the equation $2x + 3y = 18$.

a) $(-5, 7)$

b) $(3, 4)$

Solution We substitute the ordered pair into the equation and determine whether the resulting equation is true.

a)
$$\begin{array}{r|l} 2x + 3y = 18 & \\ \hline 2(-5) + 3(7) \stackrel{?}{} 18 & \text{We substitute } -5 \text{ for } x \text{ and } 7 \text{ for } y \text{ (alphabetical order).} \\ -10 + 21 & \\ 11 \mid 18 & \text{FALSE} \end{array}$$

The equation $11 = 18$ is false, so $(-5, 7)$ is not a solution.

*Here the notation $(-3, 5)$ represents an ordered pair. This notation can also represent an open interval. See Just-In-Time 6 review. The context in which the notation appears usually makes the meaning clear.

b)

$$\begin{array}{r|l} 2x + 3y = 18 & \\ \hline 2(3) + 3(4) \stackrel{?}{=} 18 & \text{We substitute 3 for } x \text{ and 4 for } y. \\ 6 + 12 & \\ 18 \quad 18 & \text{TRUE} \end{array}$$

The equation $18 = 18$ is true, so $(3, 4)$ is a solution.

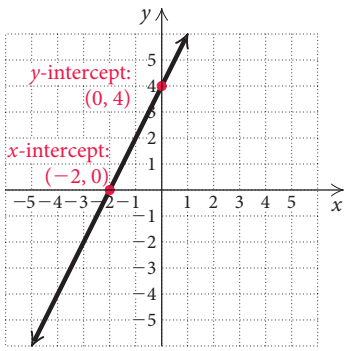
Now Try Exercise 11.

Graphs of Equations

The equation considered in Example 2 actually has an infinite number of solutions. Since we cannot list all the solutions, we will make a drawing, called a **graph**, that represents them. On the following page are some suggestions for drawing graphs.

TO GRAPH AN EQUATION

To **graph an equation** is to make a drawing that represents the solutions of that equation.



Graphs of equations of the type $Ax + By = C$ are straight lines. Many such equations can be graphed conveniently using intercepts. The ***x*-intercept** of the graph of an equation is the point at which the graph crosses the x-axis. The ***y*-intercept** is the point at which the graph crosses the y-axis. We know from geometry that only one line can be drawn through two given points. Thus, if we know the intercepts, we can graph the line. To ensure that a computational error has not been made, it is a good idea to calculate and plot a third point as a check.

***x*- AND *y*-INTERCEPTS**

An ***x*-intercept** is a point $(a, 0)$. To find a, let $y = 0$ and solve for x.

A ***y*-intercept** is a point $(0, b)$. To find b, let $x = 0$ and solve for y.

EXAMPLE 3 Graph: $2x + 3y = 18$.

Solution The graph is a line. To find ordered pairs that are solutions of this equation, we can replace either x or y with any number and then solve for the other variable. In this case, it is convenient to find the intercepts of the graph. For instance, if x is replaced with 0, then

$$\begin{aligned} 2 \cdot 0 + 3y &= 18 \\ 3y &= 18 \\ y &= 6. \end{aligned} \qquad \text{Dividing by 3 on both sides}$$

Thus, $(0, 6)$ is a solution. It is the *y-intercept* of the graph. If y is replaced with 0, then

$$\begin{aligned} 2x + 3 \cdot 0 &= 18 \\ 2x &= 18 \\ x &= 9. \end{aligned}$$

Dividing by 2 on both sides

Thus, $(9, 0)$ is a solution. It is the *x-intercept* of the graph. We find a third solution as a check. If x is replaced with 3, then

$$\begin{aligned} 2 \cdot 3 + 3y &= 18 \\ 6 + 3y &= 18 \\ 3y &= 12 \\ y &= 4. \end{aligned}$$

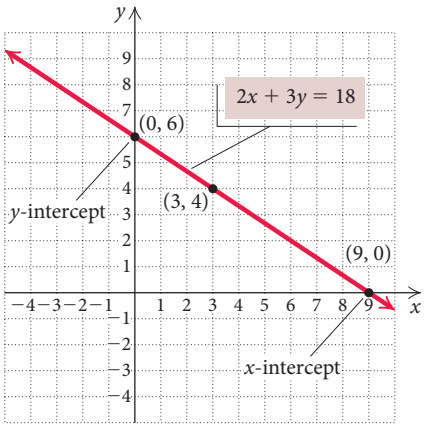
Subtracting 6 on both sides

Dividing by 3 on both sides

Thus, $(3, 4)$ is a solution.

We list the solutions in a table and then plot the points. Note that the points appear to lie on a straight line.

x	y	(x, y)
0	6	$(0, 6)$
9	0	$(9, 0)$
3	4	$(3, 4)$



Were we to graph additional solutions of $2x + 3y = 18$, they would be on the same straight line. Thus, to complete the graph, we use a straight-edge to draw a line, as shown in the figure. This line represents all solutions of the equation. Every point on the line represents a solution; every solution is represented by a point on the line.

Now Try Exercise 17.

When graphing some equations, it is convenient to first solve for y and then find ordered pairs. We can use the addition and multiplication principles to solve for y.

Suggestions for Drawing Graphs

1. Calculate solutions and list the ordered pairs in a table.
2. Use graph paper.
3. Draw axes and label them with the variables.
4. Use arrows on the axes to indicate positive directions.
5. Scale the axes; that is, label the tick marks on the axes. Consider the ordered pairs found in part (1) above when choosing the scale.
6. Plot the ordered pairs, look for patterns, and complete the graph. Label the graph with the equation being graphed.

JUST IN TIME 14

EXAMPLE 4 Graph: $3x - 5y = -10$.

Solution We first solve for y:

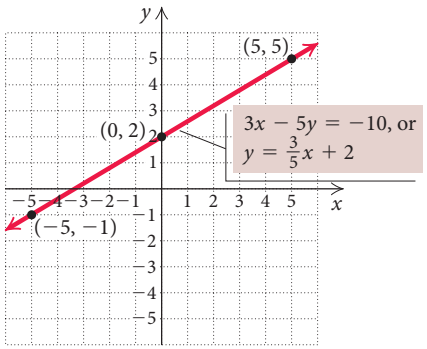
$$\begin{aligned} 3x - 5y &= -10 \\ -5y &= -3x - 10 && \text{Subtracting } 3x \text{ on both sides} \\ y &= \tfrac{3}{5}x + 2. && \text{Multiplying by } -\tfrac{1}{5} \text{ on both sides} \end{aligned}$$

By choosing multiples of 5 for x, we can avoid adding and subtracting fraction values when calculating y. For example, if we choose -5 for x, we get

$$y = \tfrac{3}{5}x + 2 = \tfrac{3}{5}(-5) + 2 = -3 + 2 = -1.$$

The following table lists a few points. We plot the points and draw the graph.

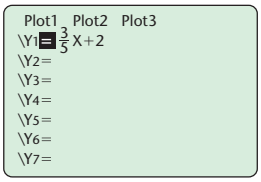
x	y	(x, y)
-5	-1	$(-5, -1)$
0	2	$(0, 2)$
5	5	$(5, 5)$

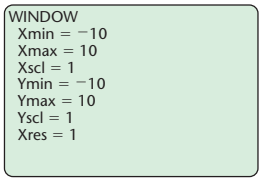


Now Try Exercise 29.

In the equation $y = \frac{3}{5}x + 2$ in Example 4, the value of y *depends* on the value chosen for x, so x is said to be the **independent variable** and y the **dependent variable**.

Technology Connection

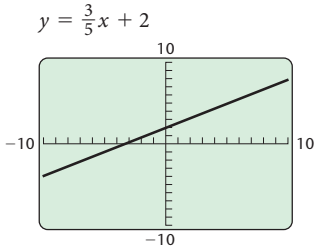




We can graph an equation on a graphing calculator. Many calculators require an equation to be entered in the form "$y =$." In such a case, if the equation is not initially given in this form, it must be solved for y before it is entered in the calculator. For the equation $3x - 5y = -10$ in Example 4, we enter $y = \frac{3}{5}x + 2$ on the equation-editor, or $y =$, screen in the form $y = (3/5)x + 2$, as shown in the window at left.

Next, we determine the portion of the xy-plane that will appear on the calculator's screen. That portion of the plane is called the **viewing window**.

The notation used in this text to denote a window setting consists of four numbers $[\text{L}, \text{R}, \text{B}, \text{T}]$, which represent the **L**eft and **R**ight endpoints of the x-axis and the **B**ottom and **T**op endpoints of the y-axis, respectively. The window with the settings $[-10, 10, -10, 10]$ is the **standard viewing window**. On some graphing calculators, the standard window can be selected quickly using the ZSTANDARD feature from the ZOOM menu.



Xmin and Xmax are used to set the left and right endpoints of the x-axis, respectively; Ymin and Ymax are used to set the bottom and top endpoints of the y-axis, respectively. The settings Xscl and Yscl give the scales for the axes. For example, Xscl $=$ 1 and Yscl $=$ 1 means that there is 1 unit between tick marks on each of the axes. In this text, scaling factors other than 1 will be listed by the window unless they are readily apparent.

After entering the equation $y = (3/5)x + 2$ and choosing a viewing window, we can then draw the graph shown at left.

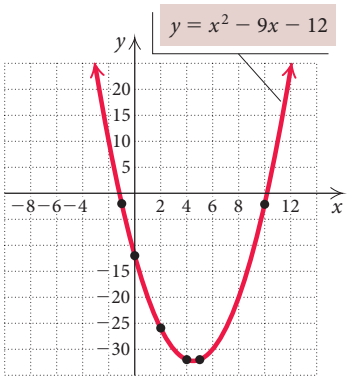
JUST IN TIME 7

EXAMPLE 5 Graph: $y = x^2 - 9x - 12$.

Solution Note that since this equation is not of the form $Ax + By = C$, its graph is not a straight line. We make a table of values, plot enough points to obtain an idea of the shape of the curve, and connect the points with a smooth curve. It is important to scale the axes to include most of the ordered pairs listed in the table. Here it is appropriate to use a larger scale on the y-axis than on the x-axis.

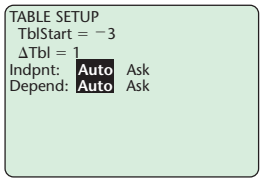
x	y	(x, y)
−3	24	(−3, 24)
−1	−2	(−1, −2)
0	−12	(0, −12)
2	−26	(2, −26)
4	−32	(4, −32)
5	−32	(5, −32)
10	−2	(10, −2)
12	24	(12, 24)

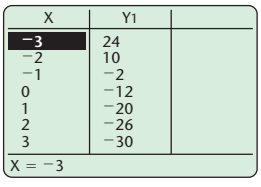
① **Select values for x.**
② **Compute values for y.**



Now Try Exercise 39.

Technology Connection



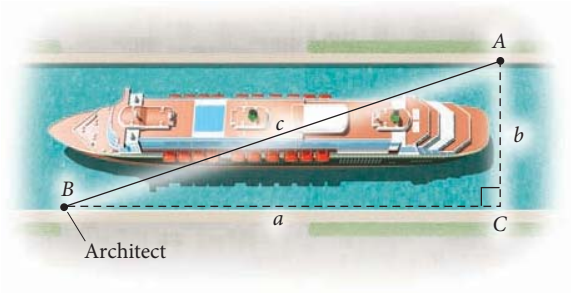


A graphing calculator can be used to create a table of ordered pairs that are solutions of an equation. For the equation in Example 5, $y = x^2 - 9x - 12$, we first enter the equation on the equation-editor screen. Then we set up a table in AUTO mode by designating a value for TBLSTART and a value for ΔTBL. The calculator will produce a table starting with the value of TBLSTART and continuing by adding ΔTBL to supply succeeding x-values. For the equation $y = x^2 - 9x - 12$, we let TBLSTART $= -3$ and ΔTBL $= 1$. We can scroll up and down in the table to find values other than those shown here.

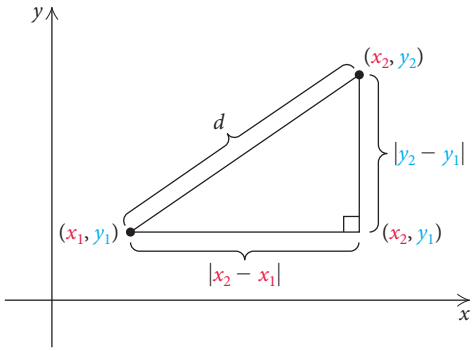
The Distance Formula

Suppose that an architect must determine the distance between two points, A and B, on opposite sides of a lane of the Panama Canal. One way in which he or she might proceed is to measure two legs of a right triangle that is situated as shown in the following figure. The Pythagorean equation, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the legs, can then be used to find the length of the hypotenuse, which is the distance from A to B.

The $5.25 billion expansion of the Panama Canal will soon double its capacity. A third canal lane is scheduled to open in 2015. (Source: *Panama Canal Authority*)



A similar strategy is used to find the distance between two points in a plane. For two points (x_1, y_1) and (x_2, y_2), we can draw a right triangle in which the legs have lengths $|x_2 - x_1|$ and $|y_2 - y_1|$.



JUST IN TIME 3, 25

Using the Pythagorean equation $c^2 = a^2 + b^2$, we have

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2.$$

Substituting d for c, $|x_2 - x_1|$ for a, and $|y_2 - y_1|$ for b in the Pythagorean equation

Because we are squaring, we can use parentheses to replace the absolute-value symbols:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

Taking the principal square root, we obtain the distance formula.

THE DISTANCE FORMULA

The **distance d** between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The subtraction of the x-coordinates can be done in any order, as can the subtraction of the y-coordinates. Although we derived the distance formula by considering two points not on a horizontal line or a vertical line, the distance formula holds for *any* two points.

JUST IN TIME 4, 22

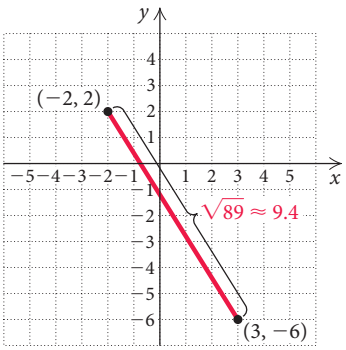
EXAMPLE 6 Find the distance between each pair of points.

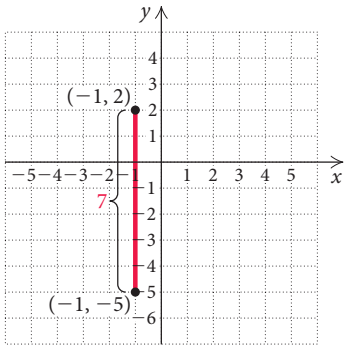
a) $(-2, 2)$ and $(3, -6)$ **b)** $(-1, -5)$ and $(-1, 2)$

Solution We substitute into the distance formula.

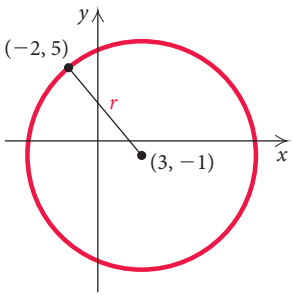
a)
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[3 - (-2)]^2 + (-6 - 2)^2} \\ &= \sqrt{5^2 + (-8)^2} = \sqrt{25 + 64} \\ &= \sqrt{89} \approx 9.4 \end{aligned}$$

b)
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-1 - (-1)]^2 + (-5 - 2)^2} \\ &= \sqrt{0^2 + (-7)^2} = \sqrt{0 + 49} \\ &= \sqrt{49} = 7 \end{aligned}$$





Now Try Exercises 41 and 49.



EXAMPLE 7 The point $(-2, 5)$ is on a circle that has $(3, -1)$ as its center. Find the length of the radius of the circle.

Solution Since the length of the radius is the distance from the center to a point on the circle, we substitute into the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{[3 - (-2)]^2 + (-1 - 5)^2}$$

Substituting r for d, $(3, -1)$ for (x_2, y_2), and $(-2, 5)$ for (x_1, y_1). Either point can serve as (x_1, y_1).

$$\begin{aligned} &= \sqrt{5^2 + (-6)^2} = \sqrt{25 + 36} \\ &= \sqrt{61} \approx 7.8. \end{aligned}$$

Rounded to the nearest tenth

The radius of the circle is approximately 7.8.

Now Try Exercise 55.